Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 92 (2016) 112 – 118

Procedia Computer Science

2nd International Conference on Intelligent Computing, Communication & Convergence

(ICCC-2016)

Srikanta Patnaik, Editor in Chief

Conference Organized by Interscience Institute of Management and Technology

Bhubaneswar, Odisha, India

A New PrecoderCodebook Design in Spatially Correlated MIMO Channels

Yin Zhu*, Chao Wang, Ping Wang, Fu-qiang Liu

School of Electronics and Information Engineering, Tongji University, Shanghai,201804, China

Abstract

In this paper, we propose a new method to design precoder codebook for spatially correlated MIMO channels. Our codebook includes a global and a local component. The codewords in the global component are uniformly distributed on the Grassmann manifold, but those in the local component are designed to center on the statistic of the optimal precoder, based on a rotation matrix codebook. Such a codebook design is applicable in both beamforming and spatial multiplexing systems. Compared with conventional approaches, it achieves good error performance with lower construction complexity.

Peer-review under responsibility of the Organizing Committee of ICCC 2016

Keywords: Spatially correlated MIMO channel; precoder codebook design; rotation matrix; spatial multiplexing;

* Corresponding author. Tel.: +86-21-69583671.
*E-mail address:*yin_zhu74@163.com

Peer-review under responsibility of the Organizing Committee of ICCC 2016
doi:10.1016/j.procs.2016.07.331

1. Introduction

In single-user multiple-input multiple-output (MIMO) systems, beamforming and spatial multiplexing techniques are used to obtain diversity and/or multiplexing gains. When channel state information (CSI) is known at transmitter, spatial multiplexing in closed-loop MIMO systems refers to the case that multiple independent data streams are precoded by a precoding matrix to be aligned to the eigen-directions of the channel, forming multiple parallel sub-channels. Beamforming, as a special case of spatial multiplexing, allows only one data stream to be precoded by a precoding vector and transmitted on the dominant eigen-direction of the channel. In practice, perfect CSI at transmitter (CSIT) is hard to obtain, especially in frequency division duplexing (FDD) systems. Thus using limited feedback to provide partial channel knowledge to the transmitter has attracted much attention in the last decade[1]. In such systems, the receiver can directly calculate the precoding matrix (vector) based on its channel knowledge. Then the precoding matrix is quantized into an element of a finite set called precoder codebook, and its index is sent to the transmitter via a low-rate feedback link. In general, the optimal precoder codebook can be computed by the generalized Lloyd algorithm according to the distribution of its elements[2]. But this method cannot adapt to channel condition in a flexible manner.

In spatially uncorrelated Rayleigh fading channels, the optimal precoder is isotropically distributed. Reference 3 points out that in a beamforming system the design of the codebook is actually a Grassmannian line packing problem. Similarly, the optimal codebook design for a spatial multiplexing system can be viewed as a Grassmannian subspace packing problem[4]. In spatially correlated MIMO channels, the optimal codebook design is dependent on the channel's statistical information. Reference 5 proposes a correlation-dependent codebook by multiplying a Grassmannian codebook with the square root of the channel's transmit correlation matrix for a beamforming system, which is shown to be a near-optimal solution. However, it is hard to apply this method in spatial multiplexing systems. In reference 6, a general structured codebook design is proposed in which a root code set is rotated and scaled to the dominant eigenvectors of the transmit correlation matrix of the channel, and it is shown to have a good performance with regard to error probability. But the codebook construction is fairly complex.

In this work, we also consider a spatially correlated MIMO channel with limited feedback. We propose a new codebook design method based on a rotation matrix codebook[7] and a relevant feedback scheme. Compared to reference 6, our codebook has a simpler construction process while still attaining good error performance.

2. System model

A single-user MIMO system with N_t transmit and N_r receive antennas is considered. M ($M \le \text{rank}(\mathbf{H}) \le \min\{N_t, N_r\}$) independent data streams are simultaneously transmitted on a narrow-band Rayleigh flat-fading spatially correlated channel. At the kth discrete-time instance, the baseband signal model can be written as $\mathbf{y}[k] = \mathbf{H}[k]\mathbf{F}[k]\mathbf{x}[k]+\mathbf{n}[k]$, where $\mathbf{H}[k]$ is the $N_r \times N_t$ channel matrix, $\mathbf{n}[k]$ is the additive white Gaussian noise with i.i.d. entries distributed according to $\mathcal{CN}(0,N_0)$, $\mathbf{F}[k]$ is the $N_t \times M$ precoding matrix, $\mathbf{x}(k) \in \mathbf{C}^{M\times 1}$ is the transmitted symbol vector satisfying $E\{\mathbf{x}[k]\mathbf{x}^H[k]\} = \frac{P}{M}\mathbf{I}_M$, P denotes the total symbol energy, $\mathbf{I}_M$ is an $M \times M$ identity matrix and $(\bullet)^H$ represents conjugate transposition. In the following we will omit k for concise expression. We constrain the structure of the precoding matrix $\mathbf{F} \in \mathcal{U}(N_t, M)$, where $\mathcal{U}(N_t,M)$ denotes a set of $N_t \times M$ matrices with orthonormal columns, i.e., $\mathbf{F}^H\mathbf{F} = \mathbf{I}_M$.

The spatially correlated MIMO channel model is given by[6]:

$$\mathbf{H} = \frac{1}{\sqrt{E\left[\text{Tr}(\mathbf{H}\mathbf{H}^H)\right]}}\mathbf{R}_r^{1/2}\mathbf{H}_{\text{i.i.d}}\mathbf{R}_t^{1/2} \tag{1}$$

wherethe entries of $\mathbf{H}_{\text{i.i.d}}$are independent and identically distributed according to $\mathcal{CN}(0,1)$. $\mathbf{R}_t = E\{\mathbf{H}^H\mathbf{H}\}$ and $\mathbf{R}_r = E\{\mathbf{H}\mathbf{H}^H\}$ represent the transmit and receive correlation matrices, respectively. In a statistical period they remain constant, but change across different periods. Furthermore, the eigen-decomposition of $\mathbf{R}_t = \mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t^H$, and the singular value decomposition (SVD) of $\mathbf{H} = \mathbf{U_H}\Sigma_\mathbf{H}\mathbf{V_H}^H$ respectively, where Λ_t and $\Sigma_\mathbf{H}$are diagonal with their elements decreasingly ordered.$\mathbf{U}_t$, $\mathbf{U_H}$ and $\mathbf{V_H}$ are unitary.We adopt a block fading model to characterize $\mathbf{H}_{\text{i.i.d}}$ which meansits elements remain constant in a block and then change independently in the next block. We also assume that ideal channel knowledge can be obtained at the receiver. At each feedback instance (one feedback interval may include several channel blocks and the number of the block takes a proper value to trade off between the feedback overhead and the system performance), the optimal precoding matrix $\mathbf{F}_{\text{opt}}$ is computed at the receiver and quantized into a codeword$\mathbf{F}_i$in a codebook $\mathcal{F}$ with size 2^B. Then the B-bit index is fed back to the transmitter via a zero-delay and error-freereverse link.

In this paper, we use the project-2 norm distance on the Grassmann manifold $d_{\text{proj2}}(\mathbf{F}_1,\mathbf{F}_2) = \|\mathbf{F}_1\mathbf{F}_1^H\text{-}\mathbf{F}_2\mathbf{F}_2^H\|_2$[4] as the distance metricto select codewords in the precoder codebook. The best codeword is the one that has the minimum proj-2 norm distance to$\mathbf{F}_{\text{opt}}$.Although the maximum-likelihood (ML) receiver is optimal for symbol decoding, it suffers from high computational complexity. Thuswe use a linear MMSE or ZF receiver for reception.

3. Feedback scheme

In a spatial multiplexing system using ZF or MMSE receiver, anoptimal precoding matrix that minimizes the symbol error rate or maximizes the mutual information of the system satisfies $\mathbf{F}_{\text{opt}}=\mathbf{V_H}(1:M)$, i.e., $\mathbf{F}_{\text{opt}}$is constituted by the first M columns of the right singular matrix $\mathbf{V_H}$ of the channel[4].

In a spatially correlated MIMO channel, the distribution of the optimal precoding matrix is determined by the channel correlation matrix $\mathbf{R}_t$. Based on this property, reference 9 studies the system performance of using $\mathbf{F}_{\text{stat}}=\mathbf{U}_t(1:\text{M})$, i.e. a matrix constituted by the M dominant eigenvectors of $\mathbf{R}_t$, as the precoding matrix. (Since $\mathbf{F}_{\text{stat}}$ represents the statistical characteristic of the optimal precoding matrix, we term this approach as statistical precoding[9].) It is shown to have good performance when $\mathbf{R}_t$ has M equal non-zero eigenvalues and $\mathbf{R}_r$ has N_r equal non-zero eigenvalues. In this case, $\mathbf{F}_{\text{opt}}$appears very close to $\mathbf{F}_{\text{stat}}$ with high probability. In the general case, however, optimal precoder is distracted from its statistic (but still centers on it) and can no longer be represented precisely only by$\mathbf{F}_{\text{stat}}$, so significant performance gap exists between using the statistical precoding matrix and the optimal precoding matrix (when perfect CSIT is available), especially in the high-SNR regime. Thus a codebook designed according to the distribution of $\mathbf{F}_{\text{opt}}$ is a good choice.

In this work, we propose to construct anew codebook which includes a global and a local component to adapt to the optimal precoder distribution. The global component is isotropically distributed on the Grassmann manifold to cover the whole quantization range. The local component is formed by perturbing $\mathbf{F}_{\text{stat}}$ with a rotation matrix codebook [7] to better quantize the precoders that center on it. The perturbing radius $r = E\{d_{\text{proj2}}(\mathbf{F}_{\text{opt}}, \mathbf{F}_{\text{stat}})\}$, which is the average project-2 norm distance between $\mathbf{F}_{\text{opt}}$and its statistic$\mathbf{F}_{\text{stat}}$. The feedback scheme is illustrated in Fig.1.

During one statistical period, a training stage is set aside at the beginning during which the statistical information of the channel including $\mathbf{F}_{\text{stat}}$and r is computed at the receiver. They arequantized and sent to the transmitter by which the same local component can be constructed at both ends. Then, the transmitter starts sending data to the receiver usingthis local component. This is termed as feedback stage, which contains many feedback instances. The channel matrix changes independently across different feedback instances. Thus at the beginning of each instance, the receiver calculates and quantizes the optimal precoding matrix $\mathbf{F}_{\text{opt}}$ before feeding the index of the codeword to the transmitter.As channel correlation changes much more slowly comparing to the instantaneous channel realization, the feedback overhead of $\mathbf{F}_{\text{stat}}$and $r$can almost be omitted.

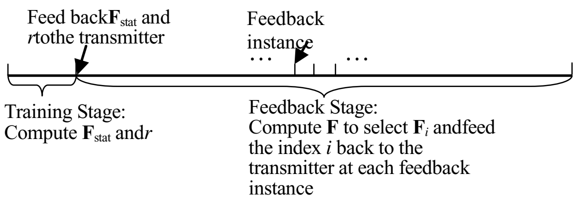

Fig.1. Feedback scheme at the receiver during one statistical period.

4. Rotation matrix based local codebook design

As mentioned above, we design a local codebook by perturbing $\mathbf{F}_{\text{stat}}$ using a rotation matrix codebook with radius r so that the local codebook is centered on $\mathbf{F}_{\text{stat}}$. Let the local codebook $\mathcal{F}_{\text{loc}} = \{\mathbf{F}_{\text{loc},i}\}_{i=1}^{L}$ where $L = 2^B$ is the size of the codebook. Define a codeword generation function $\mathcal{S}$ as following:

$$\mathcal{S} : \mathbf{C}^{N_t \times N_t} \times \mathcal{U}(N_t, M) \rightarrow \mathcal{U}(N_t, M) \tag{2}$$

The codeword of $\mathcal{F}_{\text{loc}}$ can be formed by:

$$\mathbf{F}_{\text{loc},i} = \mathcal{S}(\mathbf{F}_{\text{stat}}, r\mathbf{\Theta}_i) = proj\left(\sqrt{1-r^2}\mathbf{F}_{\text{stat}} + r\mathbf{\Theta}_i\mathbf{F}_{\text{stat}}\right), \; i=1,\ldots,L \tag{3}$$

where $\mathbf{\Theta}_i \in \mathcal{U}(N_t,N_t)$ is the element of a rotation matrix codebook $\mathcal{Q} = \{\mathbf{\Theta}_i\}_{i=1}^{L}$. The projection operator $proj(\bullet)$ ensures that $\mathbf{F}_{\text{loc},i}$ still belongs to $\mathcal{U}(N_t, M)$ after perturbing $\mathbf{F}_{\text{stat}}$. Using eq. (1) for each $i = 1, \ldots, L$, we can obtain all the codewords of $\mathcal{F}_{\text{loc}}$.

To obtain a good local codebook, it is obvious that the perturbing component $\mathbf{F}_{\text{per},i} = \mathbf{\Theta}_i\mathbf{F}_{\text{stat}}$ ($i = 1,\ldots,L$) should be distributed as uniformly as possible. Let $\mathcal{F}_{\text{per}} = \{\mathbf{F}_{\text{per},i}\}$ ($i = 1,\ldots,L$). It is required that the minimum distance between $\mathbf{F}_{\text{per},i}$ and $\mathbf{F}_{\text{per},j}$ ($i, j = 1,\ldots,L$, $i \neq j$) is maximized.

Let $\delta(\mathcal{F}_{\text{per}}) = \min_{1\le i<j\le L} d_{\text{proj2}}(\mathbf{F}_{\text{per},i}, \mathbf{F}_{\text{per},j})$. The problem converts to how to design the rotation matrix codebook $\mathcal{Q}$ tomaximize $\delta(\mathcal{F}_{\text{per}})$. From the above definition we can see:

$$\begin{aligned}
\delta(\mathcal{F}_{\text{per}}) &= \min_{\mathbf{\Theta}_i,\mathbf{\Theta}_j\in\mathcal{Q}} d_{\text{proj2}}(\mathbf{\Theta}_i\mathbf{F}_{\text{stat}}, \mathbf{\Theta}_j\mathbf{F}_{\text{stat}}) \\
&\overset{a}{\le} \min_{\mathbf{\Theta}_i,\mathbf{\Theta}_j\in\mathcal{Q}} d_{\text{chord}}(\mathbf{\Theta}_i\mathbf{F}_{\text{stat}}, \mathbf{\Theta}_j\mathbf{F}_{\text{stat}}) \\
&\overset{b}{\le} 2M\sqrt{2N_t} \min_{\mathbf{\Theta}_i,\mathbf{\Theta}_j\in\mathcal{Q}} \sqrt{1-\frac{1}{N_t}\left|\operatorname{tr}(\mathbf{\Theta}_i^H\mathbf{\Theta}_j)\right|} \\
&= 2M\sqrt{2N_t} \min_{\mathbf{\Theta}_i,\mathbf{\Theta}_j\in\mathcal{Q}} d(\mathbf{\Theta}_i, \mathbf{\Theta}_j)
\end{aligned} \tag{4}$$

The inequality (a) folowsreference 8, where d_{chord} is the chordal distance on the Grassmann manifold and is defined as:

$$d_{\text{chord}}(\mathbf{X}, \mathbf{Y}) = \sqrt{M - \left\|\mathbf{X}^H\mathbf{Y}\right\|_F^2} \tag{5}$$

The inequality(b) followsreference 7 and the distance metric

$$d(\mathbf{\Theta}_i, \mathbf{\Theta}_j) = \sqrt{1-\frac{1}{N_t}\left|\operatorname{tr}(\mathbf{\Theta}_i^H\mathbf{\Theta}_j)\right|} \tag{6}$$

is defined on the Riemannian manifold.

Let $\delta\left(\mathcal{Q}\right)=\min_{\Theta_i,\Theta_j\in\mathcal{Q}} d\left(\Theta_i,\Theta_j\right)$, then to maximize $\delta(\mathcal{F}_{\text{per}})$ we should design the rotation matrix codebook $\mathcal{Q}$to satisfy:

$$\mathcal{Q}=\arg\max_{\tilde{\mathcal{Q}}}\delta\left(\tilde{\mathcal{Q}}\right) \tag{7}$$

According to above criterion, $\mathcal{Q}$ can be obtained offline using vector quantization method[2].

Using a rotation matrix codebook as a perturbation to construct a differential codebook for a time-correlated MIMO channel was proposed in reference 7. It was assumed that when user speed is low, there exists a strong correlation between the optimal precoding matrices across consecutive feedback instances, i.e., at any feedback instance the current precoding matrix distributes around the previous one with a small radius. Thus the current codebook can be constructed by perturbing the previous precoder with that radius. We adopt a similar idea in this paper, but the application scenario is completely different. More specifically, we aim to build efficient precoder codebooks in spatially correlated rather than time correlated MIMO channels. The correlation that needs to be focused occurs between the actual optimal precoding matrix and its statistics, instead of between two precoding matrices. Thus the designing criterion is established based on the distance metric presented in eq. (2).

So far, we have constructed a local codebook (the local component) that covers the range in which $\mathbf{F}_{\text{opt}}$ appears with high probability, i.e., the range centered on $\mathbf{F}_{\text{stat}}$ with radius r. But there is still a low probability that $\mathbf{F}_{\text{opt}}$ scatters on other positions in the Grassmann manifold. We use a uniformly distributed Grassmannian codebook (the global component) to quantize those precoders. Thus the two components together form the whole codebook. The optimal way of properly assigning feedback rates to the two codebooks is in general hard to find. However, in the next section we will show through computer simulations that simply allocating each component with $B/2$ bits would already lead to good error performance.

5. Simulations and conclusion

In this section, we use simulation results to verify the performance of the proposed codebook design.Although our methodis applicable in any spatial correlated MIMO channel, we assume linear arrays at the transmitter and the receiver, which is also a commonly used deployment in practical systems, to simplify the simulation parameters. In addition, we assumethe receive correlation matrix$\mathbf{R}_r=\mathbf{I}_{N_r}$ for convenience as it has been shown in reference 6 that the system performance is mainly influenced by the transmit correlation matrix $\mathbf{R}_t$.This case also corresponds to a rich scattering environment at the receiver and it is a reasonable assumption in a practical system.Let$N_t = N_r = 4$. The transmit correlation matrix can be modeled as an exponential matrix[10]:

$$R_t=\begin{bmatrix} 1 & t & t^2 & t^3 \\ t^* & 1 & t & t^2 \\ \left(t^*\right)^2 & t^* & 1 & t \\ \left(t^*\right)^3 & \left(t^*\right)^2 & t^* & 1 \end{bmatrix}$$

where$t$ is the complex correlationcoefficient and t^* is its conjugate.The global andthe localcomponents are allocated with the same feedback rate, i.e. they have identical size $2^{B/2}$. We consider two cases with the number of data streams$M$ = 1 and $M = 2$.

Fig.2 and Fig.3 show the bit error rate (BER) comparisons of our proposed scheme with some conventional approaches, in systems with QPSK modulation, ZF receiver, and total feedback rate B=32 bits. The curve marked with perfect CSIT denotes the performance that the exact optimal precoding matrix at each feedback period is known at the transmitter. From Fig. 2, we can see the performance of the proposed method is very close to that with codebook proposed in reference 5, which is known as a codebook design that leads to near-optimal performance in MIMO beamforming systems. Thus as expected, the BER curves of both these two methods are close to that with perfect CSIT. For the statistical precoding feedback approach proposed in reference 9, as mentioned in Section 3, there is a significant gap between its performance and that with perfect CSIT in the large SNR region. From Fig.3, we can see our codebook also brings a good error performance, comparing to the case with perfect CSIT.

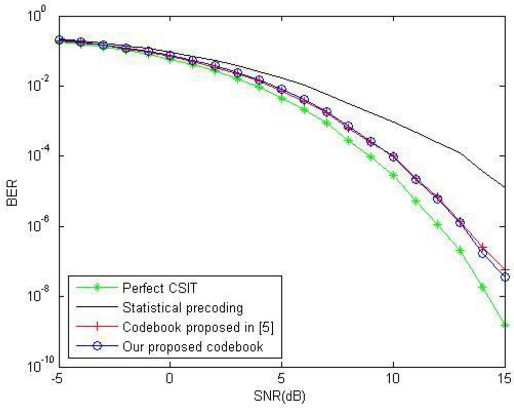

Fig.2. BER comparison of different codebooks(M = 1, $|t|$ = 0.5).

Note that we do not directly compare the performance of our codebook to that proposed in reference 6, due to its very high complexity in codebook construction. However, it is known that in a MIMO beamforming system, the codebooks proposed in reference 5 and 6 have a similar performance. From the results shown in Fig.2, it can be concluded that, compared with the codebook designed in reference 6, our method can lead to good error performance with smaller construction complexity. The similar result also applies to spatial multiplexing systems, by analyzing Fig.3. The advantages of the codebook design proposed in this paper are thus clearly exhibited.

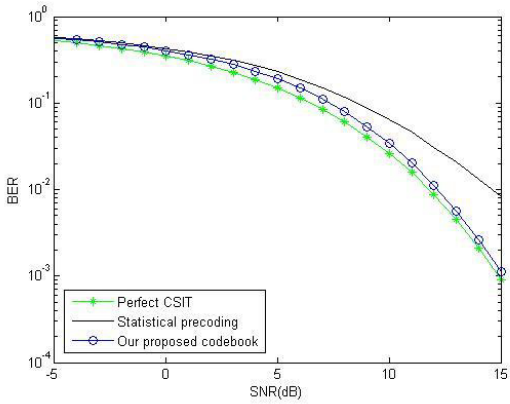

Fig.3. BER comparison of different codebooks(M = 2, $|t|$ = 0.5).

Acknowledgements

This work was supported partly bythe National Science and Technology Major Project of China under Grant2015ZX03002009-003, the Shanghai Pujiang Project under Grant 14PJ1408600,and the Fundamental Research Funds for the Central Universities (1709219004).

References

1. Love DJ,HeathJr.RW,Lau VKN,GesbertD,Rao BD,Andrews M.An Overview of Limited Feedback Wireless Communication Systems.*IEEE J. Sel. Areas Commun*. 2008;**26(8)**: 1341-1365.
2. NarulaA, Lopez MJ, TrottMD, WornellGW. Efficient Useof Side Information in Multiple-Antenna Data Transmission over Fading Channels.*IEEE J. Sel. Areas Commun*.1998;**16(8)**: 1423-1436.
3. Love DJ, Heath Jr.RW, StrohmerT. GrassmannianBeamforming for Multiple-input Multiple-output Wireless Systems.*IEEE Trans. Inform. Theory*.2003; **49(10)**: 2735-2747.
4.Love DJ,Heath Jr.RW.Limited Feedback Unitary Precoding for Spatial Multiplexing Systems.*IEEE Trans. Inform. Theory*.2005;**51(8)**: 2967-2976.
5.Love DJ,Heath JrRW. GrassmannianBeamforming on Correlated MIMO Channels.*in Proc. IEEE GLOBECOM*.2004;**1**: 106-110.
6. RaghavanV, SayeedAM, VeeravalliV. Limited Feedback Precoder Design for Spatially Correlated MIMO Channels.*in Proc. Conf. Info. Sciences and Systems*.Mar. 2007, p. 113-118.
7. Kim T, Love DJ, ClerckxB. MIMO Systems with Limited Rate Differential Feedback in Slowly Varying Channels.*IEEE Trans. Commun.* 2011;**59(4)**: 1175-1189.
8.Edelman A,Arias TA, SmithST. The Geometry of Algorithmswith OrthonormalityConstraints.*SIAM J. Matrix Anal. Appl.* 1999;**20(2)**: 303-353.
9.RaghavanV, SayeedAM,VeeravalliVV. Semiunitary Precoding for Spatially Correlated MIMO Channels.*IEEE Trans. Inform. Theory*.2011; **57(3)**: 1284-1298.
10.ClerckxB,Kim G,KimSJ.Correlated Fading in Broadcast MIMO Channels:CurseorBlessing?*inProc. IEEE GLOBECOM*.2008, p. 1-5.